# Discussion 13 Worksheet

## Double integrals in polar coordinates and surface areas of graphs

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MATH 53 Multivariable Calculus

## 1 Double integral practice

Compute these integrals:

(a) $\iint_D x \cos y dA$ where $D$ is bounded by $y = 0, y = x^2, x = 1$;

(b) $\iint_D 2x - y dA$ where $D$ is bounded by the circle with center at the origin and radius 2.

(c) Find the volume of the solid under the surface $z = xy$ and above the triangle with vertices $(1, 1), (4, 1), (1, 2)$.

(d) Find the volume enclosed by the "parabolic cylinders" $z = x^2, y = x^2$ and the planes $z = 0$ and $y = 4$.

(e) Find the volume of the solid by subtracting two volumes. The solid is enclosed by the parabolic cylinders $y = 1 - x^2, y = x^2 - 1$ and the planes $x + y + z = 2, 2x + 2y - z + 10 = 0$.

## 2 Polar Integration

Remember $dA$ becomes $rdrd\theta$.

(a) $\iint_D x^2 y dA$ where $D$ is the top half of the disk with center the origin and radius 5;

(b) $\iint_D e^{-x^2-y^2} dA$ where $D$ is the region bounded by the semicircle $x = \sqrt{4 - y^2}$ and the $y$-axis.

## 3 Surface Areas

Parametrize the following surfaces in an appropriate way (if they are not already parametrized) and compute their normal vectors and area.

(a) The portion of the elliptic paraboloid $z = x^2 + y^2$ lying over the unit disk.

(b) The part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

## 4 Triple Integration

Change the order of integration for these integrals. Sketching the region of integration might be helpful.

(a) Rewrite the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z) dz dy dx$$

as the equivalent iterated integral in the five other orders.

## 5 Challenge

(a) Find the volume of a right pyramid by setting up a triple integral. (Hint: Place 3 vertices on the coordinate axes and the fourth at the origin and use the plane equation.)

**Note:** These problems are taken from the worksheets for Math 53 in the Spring of 2021 with Prof. Stankova.